Phase 1: Constructing Meaning and Counting Strategies for Addition & Subtraction

In *Principles and Standards*, the term *number sense* is used freely throughout the Number and Operations standard. "As students work with numbers, they gradually develop flexibility in thinking about numbers, which is a hallmark of number sense… Number sense develops as students understand the size of numbers, develop multiple ways of thinking about it, and representing numbers, use numbers as referents, and develop accurate perceptions about the effects of operations on numbers" (NCTM, 2000, p.80). "The *meaning* attached to counting is the key conceptual idea on which all other number concepts are developed" (Van de Walle, 2007, p.122).

Constructing Meaning and Counting Strategies is a critical phase in which students develop a sense of number, the relationships between numbers, and understanding of operation. These form the foundation upon which fact acquisition can occur. The early development of number is critical for many reasons. It provides the student with a deep conceptual understanding of both number and operation. For this reason, phase 1 for addition and subtraction is in separated into two parts: Understanding of Number and Understanding of Addition and Subtraction (operations). This early development provides students with a strategy and all other relationships and strategy development is directly dependent on these understandings.

Understanding of Number

"A rich and thorough development of number relationships is a critical foundation for mastering of basic facts. Without number relationships, facts must be rotely memorized. With number understanding, facts for addition and subtraction are relatively simple extensions" (Van de Walle, 2007, p. 120).

Constructing meaning and counting strategies is about helping students connect different meanings, interpretations, and relationships to addition and subtraction so that they can effectively use these operations in real-world situations (Van de Walle, 2007). This phase involves the students developing a highly integrated understanding of operation and the many different but related meanings these operations take on in everyday contexts.

The development of numbers up to 20 can be extended to larger numbers, operations, basic facts and computations. Counting tells how many things are in a set. When counting a set of objects, the last word in the counting sequence names the quantity for that set. Numbers are related to each other through a variety of number relationships. The number 7 for example is more than 4, two less than 9, composed of 3 and 4, as well as 2 and 5, is three away from ten, and can be quickly recognized in several patterned arrangements of dots (subitizing). This knowledge will extend to an understanding of 17, 57, and 370.

Assumptions: Prior to working on the strategies below, student should have these basic abilities. Students will be able to:

- Demonstrate one-to-one correspondence
- Rote count to 20
- Attach meaning to counting (quantity, size)
- Show relationships of more, less, and same

Strategies for Constructing Meaning of Number

Read and Write Numerals 0 to 20

It is important that students can name and write numerals. Students learn to do this without attaching meaning as to the numerals value. For example, a child can write the numeral 5 and name it, but may not be able to make a set of 5 objects to represent this numeral. "Helping students read and write the single digit numerals 0 to 9 is similar to teaching them to read and write letters of the alphabet. Neither has anything to do with number concepts" (Van de Walle, 2007, p.122).

Counting Forward and Backward to 20

Although counting using a forward sequence of numbers is relatively familiar to most young children, counting on (from any number) and counting back are difficult skills. Fosnot and Dolk (2001) describe the ability to count on as a "landmark" on the path to number sense. Teachers should monitor students counting strategy. Students will eventually demonstrate the counting on strategy, when it becomes meaningful and useful (Van de Walle, 2007).

Counting Objects to 20

The idea that number means "amount" and the number that you end on when counting a set tells the amount of the entire set is difficult for children to understand. "Only the counting sequence is a rote procedure. The *meaning* attached to counting is the key conceptual idea on which all other number concepts are developed" (Van de Walle, 2007, p.122). Fosnot and Dolk (2001) make it very clear that an understanding of cardinality and the connection to counting is not a simple matter. Students will learn how to count (matching counting words with objects) before they understand the last count word indicates the amount of the set or the cardinality of the set (Van de Walle, 2007).

Make Sets of Objects Up to 20

Students should have various opportunities to create sets of objects to represent quantities up to 20. Many students come to kindergarten able to count small sets of objects; however, some children will need much practice in counting various sets of objects. Attention to the size of the sets being counted must be considered. The size of sets and the arrangements of the materials impact students' ability to count them. Using smaller sets and organized groupings provide students more success (Van de Walle, 2007).

Match a Numeral to a Set

After students have created sets of counters, they begin to match the numeral to the set to show how many. To realize the last count word indicates the amount of the set, some students may need to count each counter starting from 1. Some students may need an organizer (ex. a tens-frame) to help them count to make this match. Arrangement of counters on fives-, tens-, and double tens-frames can vary. Sometimes the organization of counters is specific, but teachers should be cautioned that a rigid organizational approach is not always appropriate. Counter arrangements vary from student to student.

Doubles and Near Doubles

Doubles and near-doubles are usually thought of as a reasoning strategy, however, they are simply special cases of the general part-part-whole construct. Students in this phase begin to understand doubles as special images and begin to develop these number relationships. For example, five fingers on each hand is a special image that can show double 5 is 10. These relationships are often considered a strategy for memorizing basic facts, but young students can begin to develop these relationships with special images or pictures (Van de Walle, 2007).

Subitize
Subitizing is "instantly seeing how many." There are 2 types of subitizing. Conceptual subitizing involves students recognizing a number pattern as a composite of parts and as the whole. Spatial patterns on dominoes are an example. Seeing a domino with 3 individual dots on each side and as "one six" is one example. Perceptual subitizing is recognizing a number without using other mathematical processes. Students may use perceptual subitizing to make units for counting and build on cardinality. Quantities up to ten can be known and named without the routine of counting. This can then aid in counting on (from a known patterned set) or learning combinations of numbers (seeing a pattern of 2 known smaller patterns). A student will name the amount on the domino above as the whole (6) without counting the dots or parts. Subitizing dot arrangements influence the level of difficulty. Students typically find rectangular arrangements easier. Linear, circular, and scramble arrangements increase the levels of difficulty (Clements, 1999).

Understanding of Addition and Subtraction

Constructing Meaning of Addition & Subtraction

Addition and subtraction are connected. When the parts of a set are known, addition is used to name the whole in terms of the parts. Addition names the whole in terms of combining the parts to find a sum and subtraction names a missing part by taking away, removing, or comparing to find a difference (Van De Walle, 2007 & Math to Learn, 2002). This simple definition serves both action situations and static or no-action situations. "A good understanding of the operations can firmly connect addition and subtraction so that subtraction facts are a natural consequence of having learned addition. For example, 12 – 5 is 7 since 5 + 7 is 12" (Van De Walle, 2007, p. 143). Understanding subtraction as "think-addition" rather than "take away" is extremely significant for mastering subtraction facts.

Role of Addition & Subtraction Problem Structures

There are different types of addition and subtraction problems that are reflected in the ways children think about and solve these problems. The classification focuses on the types of action or relationships that the problems present. Four problem structures can be identified for addition/subtraction problems: Join, Separate, Part-Part-Whole, and Compare. Join problems involve adding an element to a given set. Separate involves removing an element from a given set. Both join and separate involve action. Part-part-whole involves the relationship between a set and two subsets. Compare involves comparisons between two disjoint sets. The context of the situation and the size of the numbers can change, but the structure involving the actions and/ or relationships remain constant. There are several types of problems within each problem structure that are identified depending upon which quantity is the unknown.

Traditionally, educators have emphasized join and separate problem types. Overwhelming emphasis on the easier join and separate problems, gives students limited definitions for addition and subtraction. Addition becomes "put together" and subtraction becomes "take away." It makes solving addition and subtraction problems involving other structures more difficult. It is important that students are exposed to all forms of problems within these structures (Van de Walle, 2007).

The Role of Direct Modeling

Direct Modeling involves the child making physical representation of each quantity in the problem and performing the action or demonstrating the relationship (depending on the problem structure) involving those sets. Then students can count to solve. ***The child*** is directly modeling the numbers and action or relationship involved in the problem. Models and counters are used by young students to help them solve contextual story problems. These are thinking tools to help students understand what is happening and to keep track of the numbers in the problem. The part-part-whole structures are used for addition and subtraction contextual story problems.

The Role of Counting Strategies

Counting strategies are more efficient and abstract than working with physical objects. Students focus on the counting sequence itself and not on constructing with manipulatives and counting actual sets to solve the problem (Carpenter, Fennema, Franke, Levi, & Empson, 1999). Descriptions of the direct modeling and counting strategies with problem examples are within the addition and subtraction problem structures links.

The Role of Context

Contextual problems are important for helping students construct a deep understanding of addition and subtraction. Contextual problems are more than simply giving students story problems to solve. Fosnot and Dolk (2001) point out that story problems are often designed with little context and written to get children to do the procedures that the teacher wants them to do. "Context problems, on the other hand, are connected as closely as possible to student's lives, rather than 'school mathematics.' They are designed to anticipate and develop student's mathematical modeling of the real world" (Van de Walle, 2007, p. 146) Contextual problems should arise from student's everyday lives (for example: field trips, ballgames, classroom experiences, and community events).

All students, even very young children, should be expected to solve contextual problems. The numbers used in these problems should be aligned with the number development of the children and should be consistent with the FCPS essential curriculum. Students should be using numbers as large as they can count meaningfully. The NCTM *Principles and Standards for School Mathematics* makes clear the value of connecting addition and subtraction in various situations. "Teachers should ensure that students repeatedly encounter situations in which the same numbers appear in different contexts" (NCTM, 2000, pg. 83). For example, the numbers 2, 3, and 5 may appear in contextual problems that could be represented by 2 + 3, 3 + 2, 5 – 2, or 5 – 3… "Recognizing the inverse relationship between addition and subtraction can allow students to be flexible in using strategies to solve problems" (NCTM, 2000, pg. 83).

Caution:
Avoid the "Key Word" Strategy

The key word strategy "sends the wrong message about mathematics." Children can "ignore the meaning and structure of the problem and look for an easy way out." The most important "approach to solving contextual problems is to analyze its structure -- make sense of it" (Van de Walle, 2007, p. 160).

Too often students are taught to look for "key words" in story problems. Posting lists of key words with their corresponding meanings can cause confusion rather than clarification for many students. When problems are written in an overly simple and formulaic way this approach may appear effective, however, this can be a misleading strategy for children (Van de Walle, 2007). The key words could indicate an incorrect operation. "Many problems have no key words. A child who has been taught to rely on key words is left with no strategy" (Van de Walle, 2007, pg. 160).

Strategies for Constructing Meaning of Addition and Subtraction

More and Less by +1 and +2

Students need to develop the numeric relationships between numbers. When students count, they have no reason to reflect on the way one number is related to another. Their goal is only to match number words with objects until they reach the end of the counting sequence. Counting on (or back) one or two counts is a useful tool in constructing the idea of relationships of numbers. For example, 3 is "two more than" or "two less than" 5 (Van de Walle, 2007).

Building Meaningful Relationships to Anchors of 5 & 10

We relate a given number to other numbers, specifically 5 & 10. These relationships are especially useful in thinking about various combinations of numbers.

The most common and perhaps the most important models for this relationship are the five- and tens-frames. Notice that the fives-frame focuses on the relationship to 5, as an anchor for numbers, but does not anchor numbers to 10. The tens-frame can be an anchor for numbers related to five and/or ten. How children are using the fives-and tens- frames provides insight into student's number concept development (Van de Walle, 2007).

Part-Whole Relationships

Focusing on quantity in terms of its parts has important implications for developing number sense. Any student who has learned to count meaningfully can count out objects, but nothing in counting a set of objects will cause a student to focus on the fact that it could be made of two parts.

Traditionally teachers have focused on giving the parts and developing the whole. Missing-part activities are a special and important variation of part-part-whole activities. In a missing-part activity, children know the whole amount and use their already developed knowledge of the parts of that whole to try to tell what the covered or hidden part is. Missing part activities serve as the forerunner to subtraction concepts (Van de Walle, 2007).

Compose/Decompose

Compose/Decompose are supported by and support the previous strategies. To conceptualize a number as being made up of two or more parts is the most important relationship that can be developed about numbers. For example, 7 can be thought of as a set of 6 and a set of 1 (More and Less by +1 and +2), a set of 5 and a set of 2 (Anchors of 5 & 10), and a set of 7 with a set of 4 exposed (Part-Whole/ Missing Part).